A Stable Supergravity Dual of Non-supersymmetric Glue

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ABSTRACT

We study non-supersymmetric fermion mass and condensate deformations of the AdS/CFT Correspondence. The 5 dimensional supergravity flows are lifted to a complete and remarkably simple 10 dimensional background. A brane probe analysis shows that when all the fermions have an equal mass a positive mass is generated for all six scalar fields leaving non-supersymmetric Yang Mills theory in the deep infra-red. The geometry can also describe the supergravity background around an (unstable) fuzzy 5-brane.

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1 Introduction

The possibility that there is a string description of large N Yang Mills theory has been speculated on for many years [1]. The AdS/CFT Correspondence [2, 3, 4] represented the first concrete example of such a duality describing the $\mathcal{N}=4$ super Yang Mills (SYM) theory. In this paper we deduce a IIB supergravity background that describes $\mathcal{N}=4$ SYM deformed by mass terms for all the adjoint matter fields leaving pure non-supersymmetric Yang Mills in the deep infra-red.

In the AdS/CFT Correspondence supergravity fields behave as sources in the dual gauge theory. Expectation values of field theory operators are obtained from functional derivatives on the supergravity partition function with respect to the boundary values of the supergravity fields. It is therefore a crucial aspect of the correspondence that the supergravity partition function must be calculable in the presence of all infinitessimal sources in order that derivatives with respect to those sources are well defined. In fact, for sources which break the $\mathcal{N}=4$ theory's conformal symmetry, infinitessimal has no meaning since they become the only mass scale in the problem. It should therefore be possible to find gravity duals of deformed versions of the $\mathcal{N}=4$ SYM theory, including non-supersymmetric theories.

The technology [5, 6, 7, 8] required to deform the AdS/CFT Correspondence has already been developed. In 5d supergravity one must identify the field with the appropriate symmetry properties to play the role of the field theory source and determine its action. Solutions of the classical equations of motion can then be found and are described in the literature as renormalization group flows. The 5d supergravity backgrounds are often hard to interpret in terms of the dual field theory. Pilch and Warner [9] have developed a miraculous ansatz for lifting the 5d solutions to provide the 10d metric and dilaton. It is then possible, with some work, to solve the field equations for the remaining supergravity potentials. The resulting solutions are open to the use of brane probes [2, 10, 11, 12, 13, 14, 15] which, using the Dirac Born Infeld action, provide a direct translation between the gravity background and the field theory description.

Much of the early work on deformations has concentrated on supersymmetric theories such as the $\mathcal{N}=4$ theory on moduli space [16], $\mathcal{N}=1$ Leigh Strassler theory [17, 10], the $\mathcal{N}=2^*$ [18, 19, 20, 11, 12] and $\mathcal{N}=1^*$ theories [21, 22, 17]. More recently interest has turned to non-supersymmetric deformations of gauge/gravity duals [23, 24, 25, 26, 27, 28, 29]. Most of these papers have focused attention [25, 26, 27, 28] on deformations of more involved $\mathcal{N}=1$ supersymmetric constructions such as the Maldecena Nunez [30] and the Klebanov Strassler [31] backgrounds. These theories have discrete vacua and hence supersymmetry breaking perturbations will not result in an unstable background. The resulting backgrounds are though neccessarily more complicated than deformations of the $\mathcal{N}=4$ theory. In [29] we constructed the first complete 10d background of a non-supersymmetric deformation of $\mathcal{N}=4$ involving a mass term for a scalar operator. The resulting field theory and supergravity background shared an instability in the scalar potential. This highlights one of the most challenging problems in constructing non-supersymmetric solutions, the need to find a stable deformation.

In this paper we will deform the AdS/CFT Correspondence by including a supergravity scalar that is a source for an equal mass term for each of the four adjoint fermions of $\mathcal{N} = 4$ SYM. We solve for the 5d supergravity flows and then use Pilch and Warner's ansatz [9, 18] and the field equations to construct the full 10d background. The resulting background is remarkably simple. The stability of the solution is then tested using a brane probe. It shows that in the field theory the 6 scalars have positive masses radiatively induced by the fermion mass and hence there is no instability to the formation of a scalar vev (this means that at the 5d supergravity level there is no instability to the scalar in the 20 of $SU(4)_R$ switching on). We expect that the SO(4) symmetry acting on the fermions prevents any other elements of the 5d scalar in the 10 of $SU(4)_R$ (corresponding to the operators $\lambda_i\lambda_j$) switching on. At the level of this analysis, the background appears stable. At first sight it may seem surprising that the 10d lift has a scalar mass operator present which was not explicitly introduced at the 5d level. However, this operator is not represented by a scalar in the 5d supergravity so its presence or otherwise is not clear at the 5d level. Many of the supersymmetric deformations [10, 18, 19, 11, 12, 21, 17] implicitly assume the presence of this operator in 5d with confirmation only coming from a brane probe in 10d as we find here. The field theory the background describes is $\mathcal{N} = 4$ SYM with masses for all the matter fields leaving pure non-supersymmetric Yang Mills in the infra-red. We call this theory Yang Mills* following the nomenclature used for supersymmetric deformations of $\mathcal{N} = 4$ SYM.

This theory is hopefully of real use as an approximation to non-supersymmetric, pure Yang Mills theory. Of course since the $\mathcal{N} = 4$ theory is strongly coupled at all scales, the deformed theory is strongly coupled at the scale of the mass of the adjoint matter fields and in this respect differs. This situation is analagous to the thermalized 5d background of Witten [4] which also describes 4d non-supersymmetric Yang Mills in the infra-red. That theory has been used though to compute glueball masses [32] with some success, supporting the use of these geometries. It will be interesting in the future to compare the predictions of these two variants to begin to determine the size of systematic errors induced by the massive matter in each. The Yang Mills* deformation is a more systematic approach to obtaining non-supersymmetric Yang Mills and is more open to the introduction of quarks (the analysis [33] of probe D7 branes in anti de-Sitter (AdS) space appears a particularly fruitful approach). The thermalization trick would induce masses for the matter fields too.

The supergravity field we study is also capable of describing an equal bilinear condensate for each of the four adjoint fermions. As part of the analysis of the $\mathcal{N} = 1^*$ theory by Polchinski and Strassler [22] they showed that placing a fuzzy D5 brane in AdS induces, asymptotically, precisely this operator (see [34] for a review of that argument). These backgrounds then plausibly describe the supergravity theory induced around a fuzzy 5-brane. A fuzzy 5-brane is not stable unless there is some force to oppose the potential energy cost of non-commutative expansion. In the $\mathcal{N} = 1^*$ theory the 5-brane is polarized by a background 2-form potential (dual to the supersymmetry breaking mass). An alternative spin was put on the idea in [34] where the fuzzy expansion was supported by centrifugal force (corresponding to the presence of a chemical potential in the field theory). In our case there is no supporting force present and hence it is not surprising that the brane probe scalar potential is unbounded. The construction is unstable to the emission of commutative D3 branes.

In the next section we describe the Yang Mills* deformation in 5d supergravity. In section 3 we describe the oxidation process to 10d and then in section 4 we brane probe the solution. The full background is gathered together in the appendix for ease of reference.

2 Deformations in 5d Supergravity

According to the standard AdS/CFT Correspondence map [3, 4] each supergravity field plays the role of a source in the dual field theory. The simplist possibility is to consider non-trivial dynamics for a scalar field in the 5d supergravity theory. We only allow the scalar to vary in the radial direction in AdS with the usual interpretation that this corresponds to renormalization group running of the source. As is standard in the literature [5, 23] we look for solutions where the metric is described by

$$ds^2 = e^{2A(r)} dx^\mu dx_\mu + dr^2 \tag{1}$$

where $\mu = 0..3$ and r is the radial direction in AdS$_5$. The scalar field has a Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\lambda)^2 + V(\lambda) \tag{2}$$

There are two independent, non-zero, elements of the Einstein tensor (G_{00} and G_{rr}) giving two equations of motion plus there is the usual equation of motion for the scalar field [5]

$$\lambda'' + 4A'\lambda' = \frac{\partial V}{\partial \lambda} \tag{3}$$

$$6A'^2 = \lambda'^2 - 2V \tag{4}$$

$$-3A'' - 6A'^2 = \lambda'^2 + 2V \tag{5}$$

In fact only two of these equations are independent but it will be useful to keep track of all of them.

In the large r limit, where the solution will return to AdS_5 at first order and $\lambda \to 0$ and $V \to \frac{m^2}{2}\lambda^2$, only the first equation survives with solution

$$\lambda = ae^{-\Delta r} + be^{-(4-\Delta)r} \tag{6}$$

a and b are constants and

$$m^2 = \Delta(\Delta - 4) \tag{7}$$

a is interpreted as a source for an operator and b as the vev of that operator since e^r has conformal dimension 1.

If the solution retains some supersymmetry then the potential can be written in terms of a superpotential

$$V = \frac{1}{8}\left|\frac{\partial W}{\partial \lambda}\right|^2 - \frac{1}{3}|W|^2 \tag{8}$$

and the second order equations reduce to first order

$$\lambda' = \frac{1}{2}\frac{\partial W}{\partial \lambda}, \qquad A' = -\frac{1}{3}W \tag{9}$$

The deformation we will consider will break supersymmetry completely and therefore not have such a description.

2.1 A Fermionic Operator

Let us now make a particular choice for the scalar field we will consider. We take a scalar from the multiplet in the 10 of SO(6). These operators have been identified [21] as playing the role of source and vev for the fermionic operator $\psi_i\psi_j$ in the field theory. In particular we will chose the scalar corresponding to the operator

$$\mathcal{O} = \sum_{i=1}^{4} \psi_i\psi_i \tag{10}$$

The potential for the scalar can be obtained from the $N = 1^*$ solution of [21] by setting their two scalars equal (to be precise one must set their $m = \sqrt{3/4}\lambda$ and $\sigma = \sqrt{1/4}\lambda$ to maintain a canonically normalized kinetic term)

$$V = -\frac{3}{2}\left(1 + \cosh^2 \lambda\right) \tag{11}$$